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HEAT EXCHANGE IN FORCED CONVECTION IN A PIPE FILLED WITH GRANULAR BED: INFLUENCE OF THE LONGITUDINAL THERMAL CONDUCTIVITY

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Results of modeling of the process of heat transfer from a circular pipe filled with granular bed with allowance for the longitudinal thermal conductivity with boundary conditions of the first and second kind have been presented. The dependences for calculation of the thermal-stabilization portion and the active heat-exchange length that reflect the influence of the hydrodynamic regime of flow of the heat-transfer agent have been obtained.

Heat exchange from pipes filled with granular bed is known to be a complex multifactor process. The main problem arising in its description is taking correct account of the influence of the contact thermal resistance of the wall zone and the hydrodynamic regime of flow of the heat-transfer agent. In [1], the simple dependence

$$\mathrm{Nu_w} = 10\,(A + 0.0061\mathrm{Re\,Pr})\,, \tag{1}$$

where A = 1.6 (for heat-conducting particles) and A = 1 (for non-heat-conducting particles), has been obtained for calculation of the wall heat-exchange coefficient.

The hydrodynamics of an infiltrated granular bed exerts an influence on the heat exchange by means of several parameters: the velocity of the heat-transfer agent and the dispersion coefficients of longitudinal and radial thermal conductivities of the granular bed. The influence of the radial thermal conductivity has been studied fairly well [1–4], whereas the influence of the longitudinal thermal conductivity is virtually not understood.

In [5], an effort has been made to calculate the temperature field with allowance for the longitudinal thermal conductivity. However, it seems impossible to use the results of this paper because of the errors made in solving the corresponding boundary-value problem.

In this connection, in the present work we sought to model the process of heat exchange with allowance for the longitudinal thermal conductivity of an infiltrated granular bed in the case of a uniform distribution of the velocity of the heat-transfer agent in the cross section of a pipe.

Boundary Conditions of the First Kind. Under stationary conditions, for the equation of thermal conductivity (two-band model) with boundary condition of the first kind on the exterior pipe surface, we have the following system:

$$\lambda_{\mathrm{r}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \lambda_{\mathrm{long}}\frac{\partial^2 T}{\partial x^2} - c_{\mathrm{f}}\rho_{\mathrm{f}} u \frac{\partial T}{\partial x} = 0\,,$$

$$r = 0\,,\ \ \frac{\partial T}{\partial r} = 0\,;\ \ x = 0\,,\ \ c_{\mathrm{f}}\rho_{\mathrm{f}} u T^{\mathrm{in}} = c_{\mathrm{f}}\rho_{\mathrm{f}} u T - \lambda_{\mathrm{long}}\frac{\partial T}{\partial x}\,; \tag{2}$$

$$r = R - l\,,\ \ -\lambda_{\mathrm{r}}\frac{\partial T}{\partial r} = K\,(T - T_0)\,,$$

where $K = (1/\alpha_{\mathrm{w}} + \delta_{\mathrm{m}}/\lambda_{\mathrm{m}})^{-1}$ is the heat-transfer coefficient allowing for the thermal resistance of the wall zone $(1/\alpha_{\mathrm{w}})$ and the pipe wall $(\delta_{\mathrm{m}}/\lambda_{\mathrm{m}})$.

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In dimensionless form, system (2) will be written as

$$\frac{\partial^2\theta}{\partial (r')^2}+\frac{1}{r'}\frac{\partial\theta}{\partial r'}=\frac{\partial\theta}{\partial \mathrm{Fo}}-\frac{1}{\mathrm{Pe}^2}\frac{\partial^2\theta}{\partial \mathrm{Fo}^2}\,,$$

$$r'=0\,,\quad \frac{\partial\theta}{\partial r'}=0\,;\quad \mathrm{Fo}=0\,,\quad \theta=1+\frac{1}{\mathrm{Pe}^2}\frac{\partial\theta}{\partial \mathrm{Fo}}\,;\qquad r'=1-\frac{l}{R}\,,\quad -\frac{\partial\theta}{\partial r'}=\mathrm{Bi}\theta\,. \tag{3}$$

Here $\theta = (T-T_0)/(T^{\text{in}}-T_0)$.

The solution of (3) was obtained by the method of integral Hankel transformation [6]:

$$\theta=\sum_{n=1}^{\infty}A_nJ_0(\mu_n r')\exp(s_n\mathrm{Fo})\,, \tag{4}$$

where

$$A_n=\frac{2J_1(\mu_n)}{\mu_n\left(J_0^2(\mu_n)+J_1^2(\mu_n)\right)\left\{1-\dfrac{s_n}{\mathrm{Pe}^2}\right\}}\,;\quad s_n=\frac{\mathrm{Pe}^2}{2}-\frac{\mathrm{Pe}^2}{2}\sqrt{1+4\frac{\mu_n^2}{\mathrm{Pe}^2}}\,;$$

μ_n are the roots of the characteristic equation $\dfrac{J_0(\mu_n)}{J_1(\mu_n)}=\dfrac{\mu_n}{\mathrm{Bi}}$.

For the heat-transfer coefficient determined by the relation

$$K_\Sigma=K\frac{T\big|_{r'=1-l/R}-T_0}{\langle T\rangle-T_0}\,, \tag{5}$$

from (4) with account for $l \ll R$ and the expression [6]

$$\int_0^1 J_0(\mu_n r')\,r'dr'=\frac{1}{\mu_n}J_1(\mu_n) \tag{6}$$

we obtain the dependence

$$K_\Sigma=K\frac{\sum\limits_{n=1}^{\infty}A_nJ_0(\mu_n)\exp(s_n\mathrm{Fo})}{2\sum\limits_{n=1}^{\infty}A_n\dfrac{J_1(\mu_n)}{\mu_n}\exp(s_n\mathrm{Fo})}\,, \tag{7}$$

which will be written in dimensionless form as

$$\mathrm{Nu}=\mathrm{Bi}\frac{\sum\limits_{n=1}^{\infty}A_nJ_0(\mu_n)\exp(s_n\mathrm{Fo})}{2\sum\limits_{n=1}^{\infty}A_n\dfrac{J_1(\mu_n)}{\mu_n}\exp(s_n\mathrm{Fo})}\,. \tag{8}$$

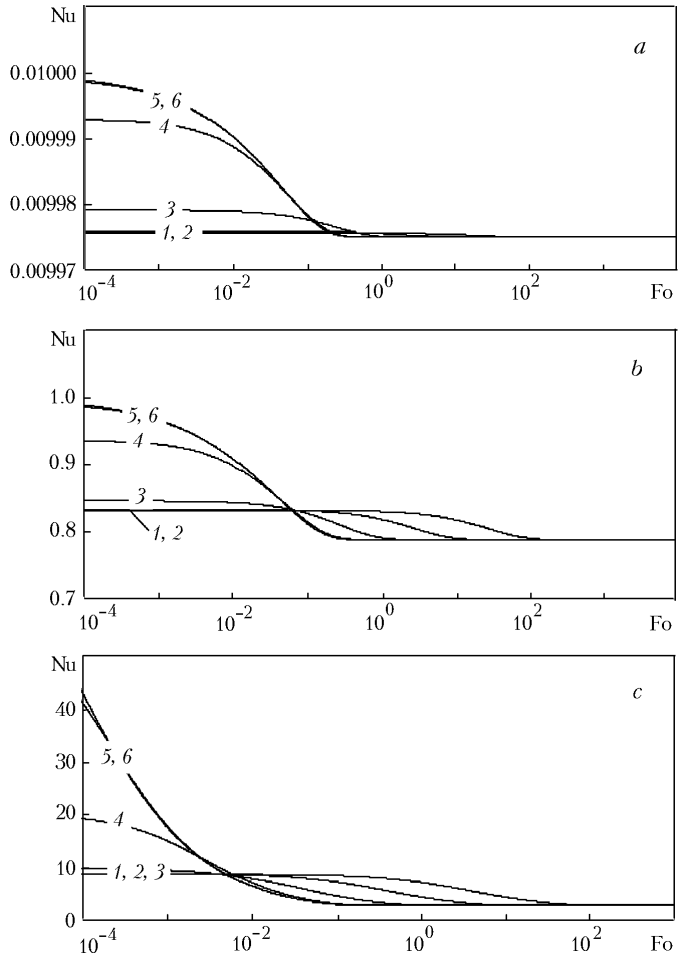

Fig. 1. Dimensionless coefficient of heat exchange in a circular pipe filled with granular bed vs. Fo number with boundary conditions of the 1st kind (calculation from (8)): a) Bi = 0.01; b) 1; c) 100 (1) Pe = 0.01; 2) 0.1; 3) 1; 4) 10; 5) 100; 6) ∞.

To evaluate the influence of the longitudinal thermal conductivity on the heat-transfer coefficient we constructed the plots Nu = f(Fo) for different Bi and Pe numbers (Fig. 1). The steady-state value of Nu is seen to be independent of Pe. Calculation for the case Pe = ∞ (absence of the longitudinal thermal conductivity) was carried out from the formula yielded by (8):

$$\mathrm{Nu}^* = \mathrm{Bi}\,\frac{\sum\limits_{n=1}^{\infty} B_n J_0(\mu_n)\exp(-\mu_n^2 \mathrm{Fo})}{2\sum\limits_{n=1}^{\infty} B_n \dfrac{J_1(\mu_n)}{\mu_n}\exp(-\mu_n^2 \mathrm{Fo})}\,, \tag{9}$$

where $B_n = \dfrac{2J_1(\mu_n)}{\mu_n\,(J_0^2(\mu_n) + J_1^2(\mu_n))}$.

We note that expression (9) is also yielded by the solution of the corresponding problem in [6], where the longitudinal thermal conductivity has been disregarded. The series in (9) rapidly converge; therefore, for Fo > 0.5 (this corresponds to $x > 0.5 c_f \rho_f u R^2/\lambda_r$), restricting ourselves to their first terms, we obtain

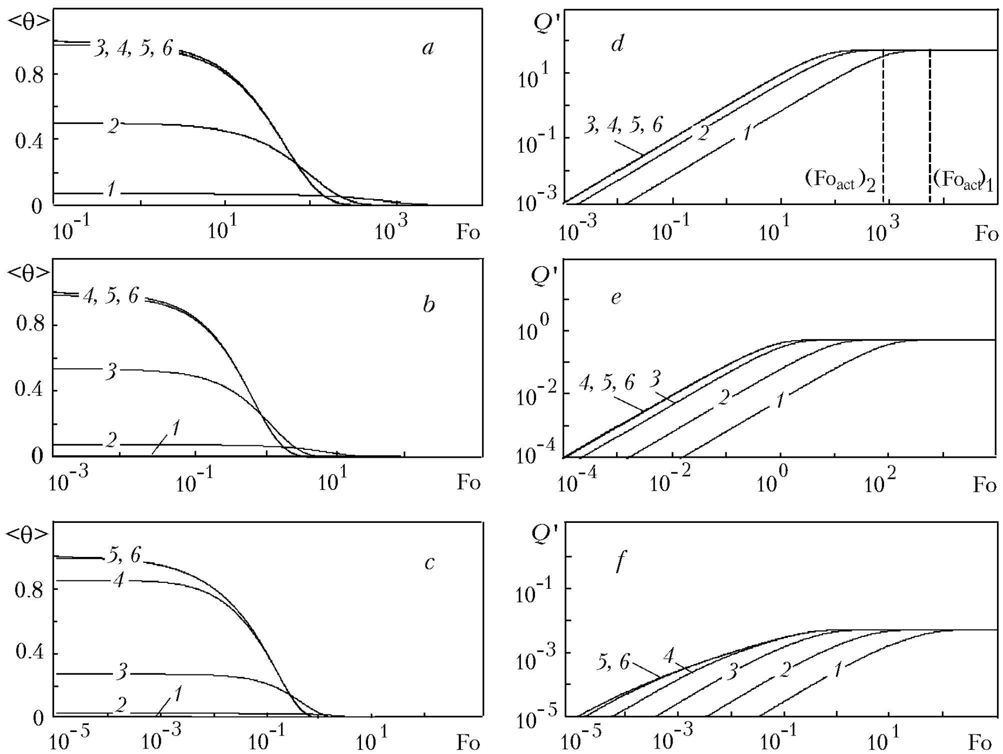

Fig. 2. Average dimensionless temperature and total dimensionless heat flux vs. Fo number (a–c, calculation from (13), d–f, from (14)): a and d) Bi = 0.01; b and e) 1; b and d) 1; c and f) 100. Notation 1–6 is the same as in Fig. 1.

$$\mathrm{Nu}^* = \frac{\mu_1^2}{2} = \frac{1}{\dfrac{1}{\mathrm{Bi}} + \dfrac{1}{2.8915}} . \tag{10}$$

In deriving (10), we have used the relation [1]

$$\mu_1 = \sqrt{\frac{2\mathrm{Bi}}{1 + \mathrm{Bi}/2.8915}} . \tag{11}$$

When the Fo numbers are large, we may restrict ourselves to the first term in the series of formula (8), and it will also be reduced to (10).

To calculate the length of the inlet portion (thermal-stabilization portion) we have obtained, on the basis of a numerical analysis of the dependences Nu = f(Fo, Pe, Bi), the following formulas:

$$\mathrm{Bi} = 0.01 \ldots 1 \quad x^* = \frac{R^2 c_f \rho_f u}{\lambda_r} (0.27\mathrm{Bi}^{0.06} + 2.33\mathrm{Bi}^{0.07}\mathrm{Pe}^{-1}) ,$$

$$\mathrm{Bi} = 1 \ldots 10 \quad x^* = \frac{R^2 c_f \rho_f u}{\lambda_r} (0.27\mathrm{Bi}^{0.06} + 2.19\mathrm{Bi}^{-0.07}\mathrm{Pe}^{-1}) ,$$

$$\mathrm{Bi} = 10 \ldots 100 \quad x^* = \frac{R^2 c_f \rho_f u}{\lambda_r} (0.43\mathrm{Bi}^{-0.15} + 2.19\mathrm{Bi}^{-0.07}\mathrm{Pe}^{-1}) . \tag{12}$$

The dimensionless temperature average over the cross section is determined as

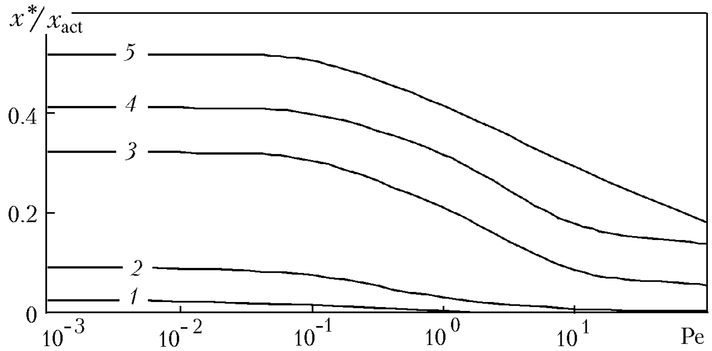

Fig. 3. Dependence of x^*/x_{act} on the Pe number: 1) Bi = 0.01; 2) 0.1; 3) 1; 4) 10; 5) 100.

$$\langle\theta\rangle = \frac{\langle T\rangle - T_0}{T^{in} - T_0} = 2\sum_{n=1}^{\infty} A_n \frac{J_1(\mu_n)}{\mu_n} \exp(s_n \text{Fo}) . \tag{13}$$

The influence of the longitudinal thermal conductivity on the average dimensionless temperature is shown in Fig. 2a–c. It is seen that we have a pronounced cooling of the heat-transfer agent even for small Fo with decrease in Pe, which is apparently attributable to the influence of the longitudinal dispersion heat flux.

The dimensionless heat flux is calculated from the formula

$$Q' = \frac{Q}{2\pi K (T^{in} - T_0) \dfrac{R^3 c_f \rho_f u}{\lambda_r}} = \sum_{n=1}^{\infty} A_n \frac{J_0(\mu_n)}{s_n} \exp(s_n \text{Fo}) - \sum_{n=1}^{\infty} A_n \frac{J_0(\mu_n)}{s_n} . \tag{14}$$

Figure 2d–f shows the quantity Q' as a function of the governing factors. It should be noted that the total heat flux removed from the heat exchanger for Fo $\to \infty$ is independent of Pe. This quantity is, apparently, determined from the balance relation $Q_\infty = \pi R^2 c_f \rho_f u (T^{in} - T_0)$, which, in dimensionless form, will be $Q'_\infty = 1/(2\text{Bi})$. The active heat-exchanger length, within which we have the cooling, increases with decrease in Pe (see Fig. 2d–f). This quantity is found from the condition

$$(Q' - Q'_\infty)/Q'_\infty \le 0.0001 . \tag{15}$$

On the basis of a numerical analysis of the dependences $Q' = f(\text{Fo, Pe, Bi})$, to calculate the active heat-exchanger length we have obtained the following formulas:

$$\text{Bi} = 0.01 \ldots 1 \quad x_{act} = \frac{R^2 c_f \rho_f u}{\lambda_r} (5.14\text{Bi}^{-0.97} + 7.19\text{Bi}^{-0.48}\text{Pe}^{-1}) ,$$

$$\text{Bi} = 1 \ldots 100 \quad x_{act} = \frac{R^2 c_f \rho_f u}{\lambda_r} (4.25\text{Bi}^{-0.26} + 6.68\text{Bi}^{-0.17}\text{Pe}^{-1}) . \tag{16}$$

The ratio x^*/x_{act} characterizing the intensity of heat-exchange processes is of great importance. This quantity plotted in Fig. 3 reflects the fraction of the active heat-exchanger length on which elevated coefficients of heat transfer are realized. It is seen that the ratio x^*/x_{act} decreases with increase in Pe and, conversely, increases with increase in Bi. The regularities established may be used in designing heat-exchange apparatuses and in optimizing them.

Boundary Conditions of the Second Kind. Under stationary conditions, for the equation of thermal conductivity (two-band model) with boundary condition of the second kind on the exterior pipe surface, we have the following system:

$$\lambda_r \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \lambda_{long} \frac{\partial^2 T}{\partial x^2} - c_f \rho_f u \frac{\partial T}{\partial x} = 0 ,$$

$$r = 0 , \ \ \frac{\partial T}{\partial r} = 0 ; \ \ x = 0 , \ \ c_f \rho_f u T^{in} = c_f \rho_f u T - \lambda_{long} \frac{\partial T}{\partial x} ; \tag{17}$$

$$r = R - l , \ \ -\lambda_r \frac{\partial T}{\partial r} = q = \text{const} .$$

We write (17) in dimensionless form:

$$\frac{\partial^2 \theta}{\partial (r')^2} + \frac{1}{r'} \frac{\partial \theta}{\partial r'} = \frac{\partial \theta}{\partial \text{Fo}} - \frac{1}{\text{Pe}^2} \frac{\partial^2 \theta}{\partial \text{Fo}^2} ,$$

$$r' = 0 , \ \ \frac{\partial \theta}{\partial r'} = 0 ; \ \ \text{Fo} = 0 , \ \ \theta = \frac{1}{\text{Pe}^2} \frac{\partial \theta}{\partial \text{Fo}} ; \tag{18}$$

$$r' = 1 - \frac{l}{R} , \ \ -\frac{\partial \theta}{\partial r'} = \frac{q}{\lambda_r} \frac{R}{T^{in}} = \overline{Q} .$$

Here $\theta = (T - T^{in})/T^{in}$.

The solution of (18) has been obtained by the method of integral Hankel transformation [6]:

$$\theta' = \frac{\theta}{\overline{Q}} = 2\text{Fo} + \frac{(r')^2}{2} - \frac{1}{4} + \frac{2}{\text{Pe}^2} - 2 \sum_{n=1}^{\infty} \frac{J_0 (\mu_n r')}{\mu_n^2 \left(1 - \dfrac{s_n}{\text{Pe}^2} \right) J_0 (\mu_n)} \exp (s_n \text{Fo}) , \tag{19}$$

where $s_n = \dfrac{\text{Pe}^2}{2} - \dfrac{\text{Pe}^2}{2} \sqrt{1 + 4 \dfrac{\mu_n^2}{\text{Pe}^2}}$ and μ_n are the roots of the characteristic equation $J_1(\mu_n) = 0$.

For the heat-transfer coefficient determined by the relation

$$\alpha_1 = \frac{q}{T \big|_{r'=1-l/R} - \langle T \rangle} , \tag{20}$$

with account for $l << R$ and expression (6) we obtain the dependence

$$\alpha_1 = \frac{1}{\dfrac{R}{\lambda_r} \left(\dfrac{1}{4} - 2 \displaystyle\sum_{n=1}^{\infty} \frac{\exp (s_n \text{Fo})}{\mu_n^2 \left(1 - \dfrac{s_n}{\text{Pe}^2} \right)} \right)} , \tag{21}$$

which is written in dimensionless form as

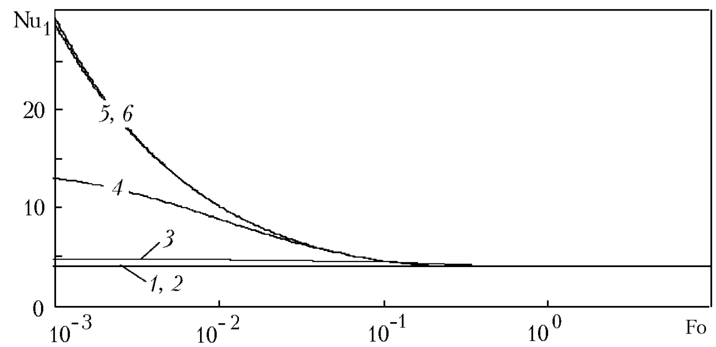

Fig. 4. Dimensionless coefficient of heat exchange in a circular pipe filled with granular bed vs. Fo number with boundary conditions of the 2nd kind (calculation from (22)). Notation 1–6 is the same as in Fig. 1.

$$\mathrm{Nu}_1 = \frac{1}{\dfrac{1}{4} - 2\sum\limits_{n=1}^{\infty} \dfrac{\exp(s_n \mathrm{Fo})}{\mu_n^2 \left(1 - \dfrac{s_n}{\mathrm{Pe}^2}\right)}} ; \qquad (22)$$

without allowance for the influence of the longitudinal thermal conductivity, it is written in the form

$$\mathrm{Nu}_1^* = \frac{1}{\dfrac{1}{4} - 2\sum\limits_{n=1}^{\infty} \dfrac{\exp(-\mu_n^2 \mathrm{Fo})}{\mu_n^2}} . \qquad (23)$$

Figure 4 shows the dependence $\mathrm{Nu}_1 = f(\mathrm{Fo})$ for different values of Pe. On the basis of an analysis of the dependence $\mathrm{Nu}_1 = f(\mathrm{Fo, Pe})$, we have obtained the following dependence for calculation of the thermal-stabilization portion:

$$x^* = \frac{R^2 c_\mathrm{f} \rho_\mathrm{f} u}{\lambda_\mathrm{r}} (0.28 + 1.45\mathrm{Pe}^{-1}) . \qquad (24)$$

We obtain $\mathrm{Nu}_1^* = 4$ for computation of the steady-state value of the heat-transfer coefficient from (22) or (23). With allowance for the resistance of the wall zone, for the total heat-transfer coefficient, we have

$$\alpha = \frac{1}{\dfrac{1}{\alpha_\mathrm{w}} + \dfrac{1}{\alpha_1}} . \qquad (25)$$

In steady-state heat exchange, we obtain

$$\alpha = \frac{1}{\dfrac{1}{\alpha_\mathrm{w}} + \dfrac{R}{4\lambda_\mathrm{r}}} . \qquad (26)$$

The dimensionless temperature average over the cross section (with allowance for the longitudinal thermal conductivity) is determined by the formula

$$\langle \theta' \rangle = 2\mathrm{Fo} + \frac{2}{\mathrm{Pe}^2} . \qquad (27)$$

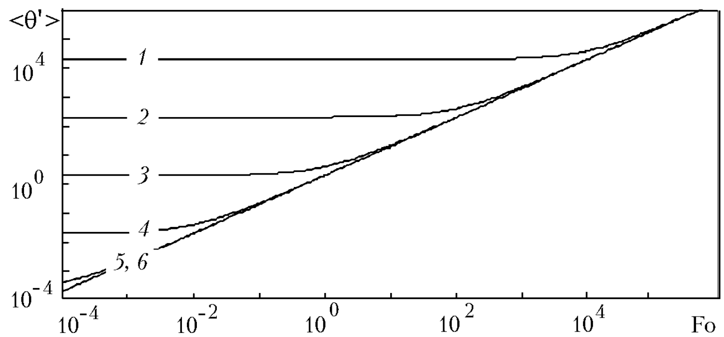

Fig. 5. Average dimensionless relative temperature vs. Fo number (calculation from (27)). Notation 1–6 is the same as in Fig. 1.

Figure 5 shows the dependence $\langle\theta'\rangle = f(\mathrm{Fo})$ for different values of Pe. It should be noted that, for low values of Pe (large coefficients λ_{long}), there are regions in which the dimensionless temperature changes only slightly. This phenomenon is caused by the influence of the dispersion heat reflux directed toward the inlet portion of the pipe $\left(-\lambda_{long}\dfrac{\partial T}{\partial x}\right)$.

CONCLUSIONS

1. The influence of the longitudinal thermal conductivity on the heat-exchange coefficient with boundary conditions of the 1st (8) and 2nd kind (22) has been established.

2. The influence of the longitudinal thermal conductivity on the size of the thermal-stabilization portion (12) and (24) has been determined.

3. The dependences for calculation of the active heat-exchanger length (16) have been found.

4. The results obtained may be used in calculating and designing heat-exchangers in the form of pipes filled with granular bed or porous packing.

NOTATION

a_f, thermal-diffusivity coefficient, m^2/sec; Bi = KR/λ_r, Biot number; c_f, heat capacity of the gas (fluid), J/(kg·K); d, particle diameter, m; Fo = $(\lambda_r x)/(R^2 c_f \rho_f u)$ and Fo$_{act}$ = $(\lambda_r x_{act})/(R^2 c_f \rho_f u)$, Fourier numbers; J_0 and J_1, Bessel functions of the 1st kind of zero and first orders; l, thickness of the wall zone, m; Nu = $K_\Sigma R/\lambda_r$, Nu$_1$ = $\alpha_1 R/\lambda_r$, and Nu$_w$ = $\alpha_w d/\lambda_f$, Nusselt numbers; Pe = $(R c_f \rho_f u)/(\lambda_{long}\lambda_r)^{1/2}$, Péclet number; Pr = ν_f/a_f, Prandtl number; $Q = 2\pi R\int_0^x q dx$, heat flux removed from the portion of length x, W; Q'_∞, total dimensionless heat flux removed from the heat exchanger; q, heat-flux density, W/m^2; Re = ud/ν_f, Reynolds number; r, radial coordinate, m; $r' = r/R$; R, pipe radius, m; T, temperature, K; $\langle T\rangle$, temperature average over the cross section x = const, K; T^{in}, temperature of the gas (fluid) at the pipe inlet, K; T_0, ambient temperature, K; u, rate of filtration of the gas (fluid), m/sec; x, longitudinal coordinate, m; x^*, pipe length on which Nu becomes stabilized (length of the inlet portion), m; x_{act}, active heat-exchanger length, m; α_w, wall coefficient of heat transfer, W/(m^2·K); α_1 and α, heat-transfer coefficients, W/(m^2·K); δ_m, thickness of the pipe wall, m; θ, dimensionless relative temperature; λ_f, thermal-conductivity coefficient of the gas (fluid), W/(m·K); λ_m, thermal-conductivity coefficient of the pipe material, W/(m·K); λ_{long} and λ_r, coefficients of longitudinal and transverse thermal conductivity of the granular bed, W/(m·K); ν_f, coefficient of kinematic viscosity,

m^2/sec; ρ_f, density of the gas (fluid), kg/m^3. Subscripts and superscripts: act, active; f, medium (gas or fluid); long, longitudinal; m, pipe material; r, radial; w, wall; Σ, total; in, inlet.

REFERENCES

1. Yu. S. Teplitskii, Heat exchange in a tube filled with granular bed, *Inzh.-Fiz. Zh.*, **77**, No. 1, 86–92 (2004).
2. M. E. Aérov, O. M. Todes, and D. A. Narinskii, Devices with a Stationary Granular Bed [in Russian], Khimiya, Leningrad (1979).
3. V. I. Mukhin and N. N. Smirnova, *Study of Heat- and Mass-Transfer Processes in Filtration in Porous Media* [in Russian], Preprint No. 26–78 of the Institute of Thermophysics, Siberian Branch of the USSR Academy of Sciences, Novosibirsk (1978).
4. R. A. Dekhtyar', D. F. Sikovskii, A. V. Gorin, and V. A. Mukhin, Heat transfer in a granular bed at moderate Reynolds numbers, *Teplofiz. Vys. Temp.*, **40**, No. 5, 748–755 (2002).
5. M. E. Aérov and N. N. Umnik, Coefficients of thermal conductivity in a granular bed, *Zh. Tekh. Fiz.*, **21**, Issue 11, 1351–1363 (1951).
6. A. V. Luikov, *Heat-Conduction Theory* [in Russian], Vysshaya Shkola, Moscow (1967).